



Editorial

Preface part 1

Journal of Approximation Theory (JAT) is devoted to advances in pure and applied approximation theory and related areas. G. G. Lorentz (February 25, 1910–January 1, 2006) is considered by many the father of Approximation Theory in North America. His 1966 book “Approximation of Functions” is a classic in the field, and was the first introduction to the subject to many in our generation. In addition, the tri-annual International Conferences of Approximation Theory that he founded in 1973 have become the most prestigious conference series of the field for the past three and a half decades. Lorentz’s unselfishness and most generous help and encouragement, particularly to young mathematicians, are remembered by all of us who had the good fortune of knowing him personally.

This is the first of a series of special issues, dedicated to our fond memory of this great mathematician. All papers, solicited by the Special Issue Editors, have undergone the same JAT editorial process, particularly in meeting the same strict journal guidelines and standards of the journal.

This issue consists of five papers on various aspects of Approximation Theory and its related areas, with the first paper by of Cantero and Simon, presenting a systematic study of the Poisson algebra induced on orthogonal polynomials on the real line and on the unit circle by some natural Poisson structures associated with the Toda and Ablowitz–Ladik hierarchies. In particular, Poisson brackets of the polynomials under investigation are derived inductively via the so-called “coefficient stripping” from the Poisson structure on coefficients of the corresponding recursion relations (or, equivalently, on matrix entries of corresponding Lax matrices), so that the resulting Poisson relations can be used to derive certain Poisson properties of the spectral data, as well as the Jacobians of the canonical coordinate changes, differential equations satisfied by the orthogonal polynomials under iso-spectral evolutions of Lax matrices, etc.

In the second paper, Goh, Goodman, and Lee observe that while the general theory and methods for the construction of wavelets and framelets in the stationary and non-stationary settings have been established, explicit construction of wavelets and framelets in most cases remains a challenging task. The authors present two constructions for tight frames for $L^2(\Omega)$, where Ω is a region in a Euclidean space, and the techniques are believed to be applicable to locally parametrizable piecewise smooth manifolds in \mathbb{R}^n , such as the unit sphere or polyhedral surfaces. The construction is based on the local matrix extension of vectors associated with transition matrices of the refinement relations across consecutive resolution levels, and hence, the number of vanishing moments could be limited to one. It is, however, applicable to both the stationary as well as the non-stationary settings.

The paper by Korevaar is devoted to the study of the long standing open problem that concerns with the (finite or infinite) number of prime pairs (i.e., for a given integer $r \geq 1$, the number of primes p for which $p + 2r$ is also a prime number). More specifically, the author is interested in the prime pair conjecture of Hardy and Littlewood, which predicts an asymptotic distribution of prime pairs with a given difference. Using the Wiener-Ikehara Tauberian theorem, the author shows that the conjecture of Hardy and Littlewood is equivalent to the boundary behavior of some function involving the complex zeros of the Riemann zeta function. A certain hypothesis on the equi-distribution of prime pairs would imply the existence of an abundance of such pairs.

The fourth paper is by Kuo, Wasilkowski, and Woźniakowski. This paper contains several new results concerning the power of function values for optimal recovery of functions in a Reproducing Kernel Hilbert Space, as compared with the power of arbitrary linear functionals. The emphasis is on the worst-case approximation errors using n function evaluations, with error measured in weighted L_2 and or L_∞ norms, where the upper bounds are obtained by constructing a rather sophisticated multi-level algorithm. It uses specially chosen quadratures to approximate linear functionals that appear in the optimal solution of the problem, when evaluation of linear functionals is allowed. The paper also examines the tractability (a concept introduced by one of the authors) of the approximation problem; i.e., the question of whether or not the constants appearing in the estimates depend polynomially on the dimension.

The paper by Schumaker, Sorokina, and Worsey addresses the question on the existence of some trivariate macro-element based on C^1 quadratic splines which can be used with arbitrary initial tetrahedral partitions. The objective of this paper is to create such an element. The resulting scheme is based on some stable local basis, and provides the full approximation power of smooth functions. The space can also be used to create globally C^1 quadratic interpolating splines for trivariate scattered data. Such splines are useful for contouring purposes in volume visualization.

As Special Issue Editors, we are most grateful to all the authors for submitting their interesting papers, and would like to express our gratitude to the referees for their conscientious work to maintain the standard of Journal of Approximation Theory. We would also like to thank the Editors-in-Chief, Paul Nevai and Amos Ron, for their encouragement and support of this project.

Charles K. Chui
Stanford University, Department of Statistics, Sequoia Hall,
390 Serra Mall, Stanford, CA 94305-4065, United States
E-mail address: cchui@stat.stanford.edu.

H.N. Mhaskar*
Department of Mathematics, California State University, Los Angeles,
5151 State University Drive, Los Angeles, CA 90032, United States
E-mail address: hnmhaskajat@gmail.com.

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* Corresponding editor.